Gen Phys:

- Get your notebook, pen/pencil, packet and worksheet from Friday
- All other materials in the cubby in front of your desk.
- Math Review test NEXT Friday!

Lesson 3

Solving Proportions

Objectives:

- Compare the size of two fractions.
- Identify a proportion
- Solve a proportion.

If two fractions have the same denominators, then it is easy to see which one is larger: just compare their numerators.

$$\frac{3}{7} < \frac{4}{7} \quad \text{because} \quad 3 < 4.$$

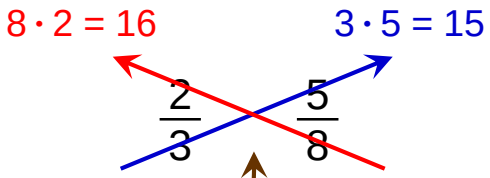
If the denominators are different, we can use the **cross product rule**:

$\frac{a}{b} < \frac{c}{d}$	if and only if $ad < bc$.
$\frac{a}{b} = \frac{c}{d}$	if and only if $ad = bc$.
$\frac{a}{b} > \frac{c}{d}$	if and only if $ad > bc$.

$$ad < bc$$

$$\frac{ad}{bd} = \frac{a}{b} < \frac{c}{d} = \frac{bc}{bd}$$

Which is larger, $\frac{2}{3}$ or $\frac{5}{8}$?



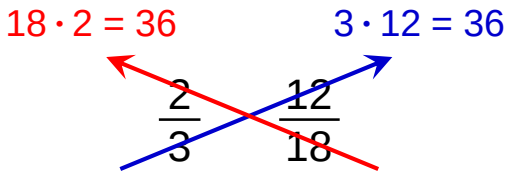
This comes from

$$\frac{2}{3} = \frac{16}{24} \quad \text{and} \quad \frac{5}{8} = \frac{15}{24}$$

Since 16 > 15, we have $\frac{2}{3} > \frac{5}{8}$.

This is why its called the cross product rule.

Which is larger, $\frac{2}{3}$ or $\frac{12}{18}$?



The cross products are both equal to 36, so $\frac{2}{3} = \frac{12}{18}$.

The **ratio** of two numbers is found by dividing the first number by the second number.

For example, the ratio of 2 to 3 is $\frac{2}{3}$.

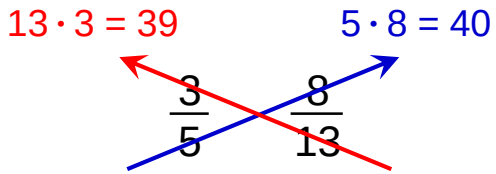
A **proportion** is an equation stating that two ratios (or fractions) are equivalent.

When we say that the fraction $\frac{2}{3}$ is equal to the fraction $\frac{12}{18}$, we are stating a proportion.

$\frac{2}{3} = \frac{12}{18}$ is a proportion.

We can use the cross product rule to determine whether a pair of ratios forms a proportion.

Example: Do the ratios $\frac{3}{5}$ and $\frac{8}{13}$ form a proportion?



Since 39 < 40, the ratios are not equal and they do not form a proportion.

When one of the numbers in a proportion is unknown, it can be found by using the cross product rule or the invariant principle for division.

Example: Solve the proportion

$$\frac{x}{20} = \frac{35}{100}$$

$\div 5$ above, $\div 5$ below

$$x = \frac{35}{5} = 7$$

Since 20 divides 100, we can easily use the invariant principle for division.

Example: Solve the proportion

$$\frac{x}{3} = \frac{7}{11}$$

$11x$ $\qquad$ $3 \cdot 7 = 21$

Since 3 does not divide 11, we will use the cross product rule.

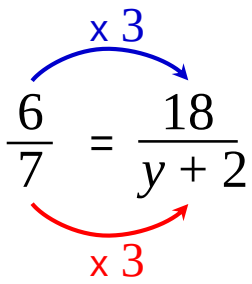
$$11x = 21$$

$$x = \frac{21}{11}$$

Example: Find the value of y that makes $\frac{6}{7}$ and $\frac{18}{y+2}$ a proportion

We want to solve the proportion $\frac{6}{7} = \frac{18}{y+2}$.

Since 6 is easily related to 18, we use the invariant principle for division.

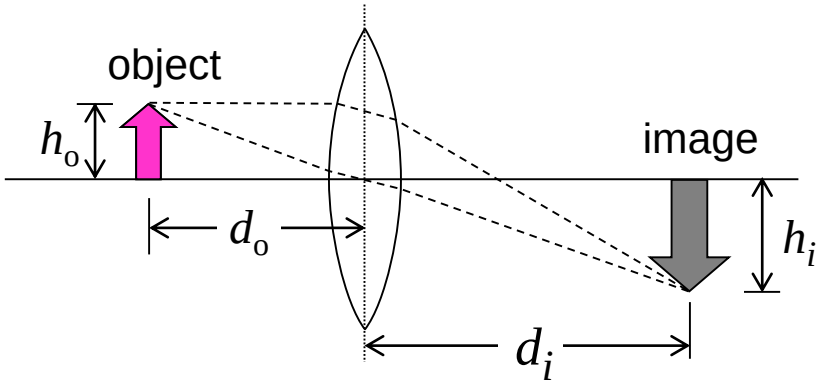


$$7 \cdot 3 = y + 2$$

$$21 = y + 2$$

$$y = 19$$

Application (Optics): If a luminous object is placed at a distance greater than the focal length away from a convex lens, then it will form an inverted image on the opposite side of the lens. The image distance d_i and height h_i are related to the object distance d_o and height h_o by the proportion



$$\frac{d_i}{h_i} = -\frac{d_o}{h_o}.$$

If d_o = 30 cm, h_o = 24 cm, and d_i = 55 cm, calculate the height of the image.

We want to solve the proportion $\frac{55}{h_i} = -\frac{30}{24}$ $\Rightarrow$ simplify $\frac{55}{h_i} = \frac{-5}{4}$ (x (–11) on both sides)

$$h_i = (4)(-11) = -44 \text{ cm}$$